FACULTY: BASIC AND APPLIED SCIENCES

DEPARTMENT: MATHEMATICS AND COMPUTER SCIENCE

SEMESTER I EXAMINATIONS (MAR. 2017)

2016 / 2017 **ACADEMIC SESSION**

COURSE CODE: MTH 203

COURSE TITLE: LINEAR ALGEBRA I

COURSE LEADER: Dr. Adelani Adesanya

DURATION: 2 Hours

HOD's SIGNATURE

INSTRUCTIONS:

1. YOU ARE TO ANSWER **FOUR** QUESTIONS OUT OF **SIX**
2. SEVERE PENALTIES APPLY FOR MISCONDUCT,CHEATING, POSSESSION OF UNAUTHORIZED MATERIALS DURING THE EXAM
3. YOU ARE NOT ALLOWED TO BORROW CALCULATORS AND WRITING MATERIALS

Q1. (a) Define "Vector space"

(b) Consider the set $v = \mathcal{R}^2$ with the standard scalar multiplication and addition defined as

$$(u_1, u_2) + (v_1, v_2) = (u_1 + 2v_1, u_2 + v_2)$$

Show that v is not a vector space.

(c) Show that $w = \{(x, y, z) \in \mathcal{R}^3 / 3x = 2y\}$ is a subspace of $\mathcal{R}^3$.

Q2. (a) Distinguish between "linear dependence and linear independence" of vectors in a vector space.

(b) Show that the following vectors $v_1 = (1, 1, 2, 1)$, $v_2 = (0, 2, 1, 1)$, $v_3 = (3, 1, 2, 0)$

form a linearly independent set.

(c) Show that the following vectors $v_1 = (1, 0, 0)$, $v_2 = (0, 1, 0)$, $v_3 = (0, 0, 1)$ *and*

$v_4 = (1, 1, 1)$ *in* $\mathbb{R}^3$ form a linearly dependent set.

Q3. . (a) Define the term 'linear Combination'

Express (i) $v_1 = (0, -26, -9)$ as a linear combination of

$v_2 = (5, 3, 7)$ *and* $v_3 = (2, -4, 1)$.

(ii) Let $v_1 = (1, 0, 1)$, $v_2 = (-1, 1, 0)$ *and* $v_3 = (1, 2, 3)$ .Express v_3 as a linear combination of v_1 *and* v_2

(b) Define "Basis and Dimension".

(c) Define term " Null Space".

Determine the Null space of the following matrix $\begin{pmatrix} 1 & -7 \\ -3 & 21 \end{pmatrix}$

Q4. (a) What do you understand by the term "Transformation"?

(b) When is a Transformation said to be linear?

(c) When is a linear transformation said to be Isomorphic?

(d) Let $T: R^2 \to R^2$ be the transformation defined by:

$T(x, y) = (x + y, x - y + 1)$. *Is* T *linear?* Justify your answer.

(e) Let V *and* W be vector spaces and let $T: V \to W$ be a linear transformation. Prove that $KerT$ is a subspace of V.

Q5. (a) Define (i) Kernel of T (ii) Nullity of T.

Find $Ker\theta$, where $\theta : \mathbb{R}^3 \dashrightarrow \mathbb{R}^3$ is defined by

$$\theta[(x_1, x_2, x_3)] = (x_1 + x_2, x_2 - x_3)$$

(b) Let $\theta: \mathbb{R}^2 \longrightarrow \mathbb{R}^1$ be defined by

$\theta[(a_1, a_2)] = {a_1}^2 + {a_2}^2$ Show that θ is not lineareven though $\theta(0) = 0$

(c) Let $L : \mathcal{R}^3 \rightarrow \mathcal{R}^2$ be defined by $L(a_1, a_2, a_3) = (a_3 - a_1, a_1 + a_2)$

(i) Compute $L(e_1)$, $L(e_2)$ and $L(e_3)$

(ii) Show that L is a linear transformation.

(iii) Show that $L(a_{1,}, a_2, a_3) = a_1 L(e_1) + a_2 L(e_2) + a_3 L(a_3)$.

Q6. Let $V = \mathcal{R}^2$ and $W = \mathcal{R}^3$. Define $L : V \rightarrow W$ by $L(X_1, X_2) = (X_1 - X_2, X_1, X_2)$

Let $F = \{(1,1), (-1,1)\}$ and let $G = \{(1,0,1), (0,1,1), (1,1,0)\}$

(a) Find the matrix representation of L using the standard bases in both V and W.
(b) Find the matrix representation of L using the standard bases in V and the basis G in W.
(c) Find the matrix representation of L using the basis F in $\mathcal{R}^2$ and the standard basis in $\mathcal{R}^3$.