ELIZADE UNIVERSITY

ILARA-MOKIN

FACULTY: BASIC AND APPLIED SCIENCES

DEPARTMENT: MATHEMATICS AND COMPUTER SCIENCE

1st SEMESTER EXAMINATION (Stream 2)

2017 / 2018 ACADEMIC SESSION

COURSE CODE: MTH 203 **COURSE TITLE:** Linear Algebra I

COURSE LEADER: Dr. A. Adesanya

DURATION: 2 Hours

HOD's SIGNATURE

INSTRUCTION:

Candidates should answer any FOUR Questions.

Students are warned that possession of any unauthorized materials in an examination is a serious offence.

Q1. (a) Define "Vector space"

(b) Consider the set $v = \mathcal{R}^2$ with the standard scalar multiplication and addition defined as $(u_1, u_2) + (v_1, v_2) = (u_1 + 2v_1, u_2 + v_2)$

Show that v is not a vector space.

(c) Show that $w = \{(x, y, z) \in \mathcal{R}^3 / 3x = 2y\}$ is a subspace of $\mathcal{R}^3$.

Q2. (a) Distinguish between "linear dependence and linear independence" of vectors in a vector space.

(b) Show that the following vectors $v_1 = (1, 1, 2, 1)$, $v_2 = (0, 2, 1, 1)$, $v_3 = (3, 1, 2, 0)$ form a linearly independent set.

(c) Show that the following vectors $v_1 = (1,0,0)$, $v_2 = (0,1,0)$, $v_3 = (0,0,1)$ and

$v_4 = (1,1,1)$ in $\mathbb{R}^3$ form a linearly dependent set.

Q3. . (a) Define the term 'linear Combination'

Express (i) $v_1 - (0,-26,-9)$ as a linear combination of

$v_2 = (5,3,7)$ and $v_3 = (2,-4,1)$.

(ii) Let $v_1 = (1,0,1)$, $v_2 = (-1,1,0)$ and $v_3 = (1,2,3)$.

Express v_3 as a linear combination of v_1 and v_2

(b) Define "Basis and Dimension". (c) Define term " Null Space".

Determine the Null space of the following matrix $\begin{pmatrix} 1 & -7 \\ -3 & 21 \end{pmatrix}$

Q4. (a) What do you understand by the term "Transformation"?

(b) When is a Transformation said to be linear?

(c) When is a linear transformation said to be Isomorphic?

(d) Let $T: R^2 \to R^2$ be the transformation defined by:

$T(x,y) = (x+y, x-y+1)$. *Is T linear?* Justify your answer.

Q5 (a) Let $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^1$ *be defined by*

$T[(a_1,a_2)] = a_1{}^2 + a_2{}^2$ *Show that T is not linear even though* $T(0) = 0$

(b) Let $L: \mathcal{R}^3 \to \mathcal{R}^2$ be defined by $L(a_1, a_2, a_3) = (a_3 - a_1, a_1 + a_2)$

(i) Compute $L(e_1)$, $L(e_2)$ and $L(e_3)$ (ii) Show that L *is a linear transformation.*

(iii) Show that $L(a_1, a_2, a_3) = a_1 L(e_1) + a_2 L(e_2) + a_3 L(a_3)$.

Q6. Let $V = \mathcal{R}^2$ and $W = \mathcal{R}^3$. *Define* $L: V \to W$ *by* $L(X_1,X_2) = (X_1 - X_2, X_1, X_2)$

Let $F = \{(1,1),(-1,1)\}$ and let $G = \{(1,0,1),(0,1,1),(1,1,0)\}$

(a) Find the matrix representation of L using the standard bases in both V and W.

(b) Find the matrix representation of L using the standard bases in

V and the basis G in W.

(c) Find the matrix representation of L using the basis

F in $\mathcal{R}^2$ *and the standard basis in* $\mathcal{R}^3$.