ELIZADE UNIVERSITY, ILARA MOKIN, NIGERIA

Mechanical Engineering Department
First Semester (2016/2017) Examination

Course Code:	ATE 403
Course Title:	Finite Element Analysis of Structures
Time Allowed:	3 Hours
Instruction:	Answer any four questions

1 a. Explain the concept of FEA and highlight five benefits of using it

b. Define node and describe with diagrams any three types of node

c. A real world model is shown in Figure 1. Sketch a possible idealized physical model to use for FEA from the real world model given

d. Solve the following system of equations by using the Gauss-Jordan elimination method.

$$\begin{cases} x + y + z = 5 \\ 2x + 3y + 5z = 8 \\ 4x + 5z = 2 \end{cases}$$

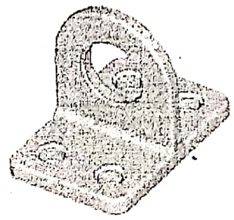

Figure 1: The "Real World" Object

2. Explain what you understand by *h-* and *p-elements* and state three differences between the two concepts.

b. State the reason why a bar element is preferred to a spring element in Finite Element Analysis and give any three assumptions inherent in the usage of a bar element.

c. Determine the eigenvalues and eigenvectors for the following matrix of equation

$$\begin{bmatrix} 8 & 4 \\ 2 & 16 \end{bmatrix}$$

3. Figure 2 depicts a tapered elastic bar subjected to an applied tensile load P at one end and attached to a fixed support at the other end. The cross-sectional area varies linearly from A_1 at the fixed support at $x = 0$ to $3A_1/4$ at $x = L$. Calculate the displacement of the end of the bar

(a) by modeling the bar as a single element having cross-sectional area equal to the area of the actual bar at its midpoint along the length, 6

(b) using two bar elements of equal length and similarly evaluating the area at the midpoint of each, and

(c) using integration to obtain the exact solution.

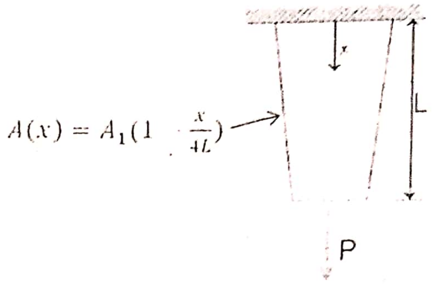

Figure 2: Tapered axial bar

4a. State Castigliano's First Theorem

b. (i) Apply Castigliano's first theorem to the system of four spring elements depicted in Figure 3 to obtain the system stiffness matrix. The vertical members at nodes 2 and 3 are to be considered rigid.

(ii) Solve for the displacements and the reaction force at node 1 of Figure 3 if

$k_1 = 8$ N/mm $\quad k_2 = 12$ N/mm $\quad k_3 = 6$ N/mm

$F_2 = -30$ N $\quad F_3 = 0 \quad F_4 = 50$ N

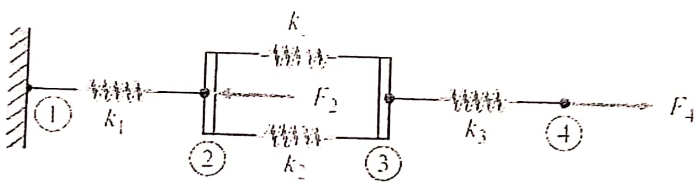

Figure 3: Four Spring Elements

5a. State the Principle of Minimum Potential Energy

b. (i) Apply the Principle of Minimum Potential Energy to the system of four spring elements depicted in Figure 3 to obtain the system stiffness matrix. The vertical members at nodes 2 and 3 are to be considered rigid.

(ii) Solve for the displacements and the reaction force at node 1 of Figure 3 if

$k_1 = 2$ N/mm $\quad k_2 = 3$ N/mm $\quad k_3 = 1.5$ N/mm

$F_2 = -30$ N $\quad F_3 = 0 \quad F_4 = 50$ N

6. a. What is the significance of flexure elements in FEA?

b. State three assumptions and restrictions underlying the development of flexure elements?

c. The nodal variables associated with a flexure element are as depicted in Figure 4 with the displacement function v(x) to be discretized expressed as $v(x) = f(v_1, v_2, \theta_1, \theta_2, x)$

subject to the boundary conditions

$$v(x = x_1) = v_1 \qquad v(x = x_2) = v_2 \qquad \left.\frac{dv}{dx}\right|_{x=x_1} = \theta_1 \quad \left.\frac{dv}{dx}\right|_{x=x_2} = \theta_2$$

Show that
$$v(x) = \lfloor N_1 \quad N_2 \quad N_3 \quad N_4 \rfloor \begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{Bmatrix} = \lfloor N \rfloor \{\delta\}$$

where N_1, N_2, N_3, and N_4 are the interpolation functions that describe the distribution of displacement in terms of nodal values in the nodal displacement vector $\{\delta\}$

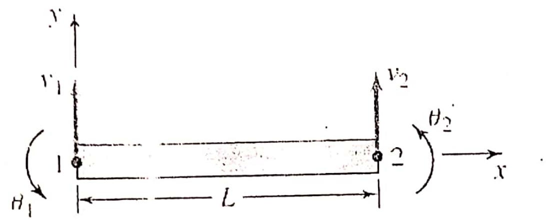

Figure 4: Beam element nodal displacements shown in a positive sense.